Illustrating Student Achievement
Using National Assessment of Educational Progress Questions:
Grade 7
Ratios and Proportional Relationships Domain

The Montana Office of Public Instruction (OPI) adopted new standards for language arts and mathematics in November 2011. The new standards will be implemented in the 2013-2014 school year with the Smarter Balanced (SBAC) assessment taking place in the spring of 2014.

This document uses National Assessment of Education Progress (NAEP) questions that seem to have a close alignment with the new standards to illustrate or suggest current levels of student achievement for the new standards. It is not intended to make any predictions about how students will do on a new assessment but may have instructional implications in terms of showing students' strengths and weaknesses. NAEP releases some items after each NAEP administration; performance data is given for the nation and states for each released item. Since 2003, every state has participated in the grade 4 and grade 8 NAEP mathematics and language arts assessments, which are given every other year. SBAC released practice tests matching the Ratios and Proportional Relationships domain have been included in this document as another example to illustrate the standards. There are no NAEP 2013 released questions as examples but these questions may be accessed via the NAEP Questions Tool (NQT).

This work has been made available through the **National NAEP Year Projects** (NNYP). This document parallels the work of Alaska's NAEP state coordinator. The following jurisdictions have made this information possible: Alaska, Iowa, New York, Florida, Oregon and the District of Columbia. For more information and resources, please visit:

- Alaska Department of Education
- Iowa Department of Education
- NYC Department of Education
- Florida Department of Education
- Oregon Department of Education
- District of Columbia
- **AIR:** Examining the Content and Context of the Common Core State Standards: A First Look at Implications for the National Assessment of Educational Progress

Montana
Office of Public Instruction
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opi.mt.gov

A note about NAEP performance: NAEP rates multiple-choice or constructed-response questions scored either right or wrong as "easy" if answered correctly by 60% or more of students, "medium" is answered correctly by 40 to 59%, or "hard" if answered correctly by fewer than 40%.

Montana Common Core Standards (MCCS):

Represent and solve problems involving multiplication and division.

- **3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

- **5.NBT.7** Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Understand ratio concepts and use ratio reasoning to solve problems.

- **6.RP.3.** Use ratio and rate reasoning to solve real-world and mathematical problems from a variety of cultural contexts, including those of Montana American Indians, , e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
  a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
  b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? As a contemporary American Indian example, it takes at least 16 hours to bead a Crow floral design on moccasins for two children. How many pairs of moccasins can be completed in 72 hours?

Apply and extend previous understandings of arithmetic to algebraic expressions.

- **6.EE.1.** Write and evaluate numerical expressions involving whole-number exponents.

Represent and analyze quantitative relationships between dependent and independent variables.

- **6.EE.9.** Use variables to represent two quantities in a real-world problem from a variety of cultural contexts, including those of Montana American Indians, that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- **7.RP.1.** Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour or apply a given scale factor to find missing dimensions of similar figures.*

Analyze proportional relationships and use them to solve real-world and mathematical problems.

- **7.RP.2.** Recognize and represent proportional relationships between quantities. Make basic inferences or logical predictions from proportional relationships.
  a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).
  b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships in real world situations.
  c. Represent proportional relationships by equations and multiple representations such as tables, graphs, diagrams, sequences, and contextual situations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.

d. Understand the concept of unit rate and show it on a coordinate plane. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

Analyze proportional relationships and use them to solve real-world and mathematical problems.

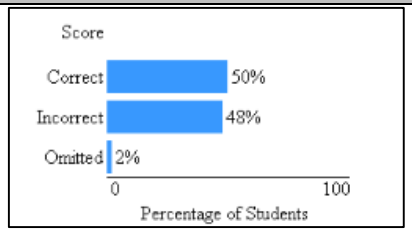
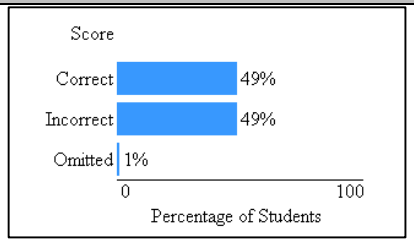
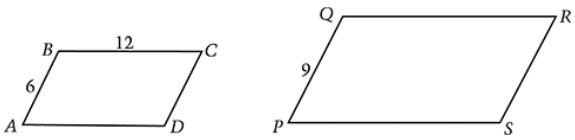
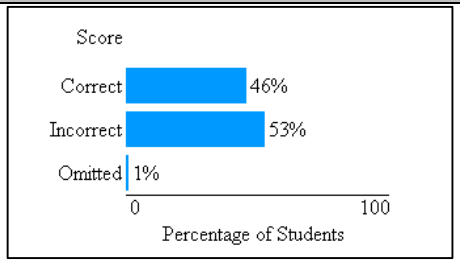
- **7.RP.3.** Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

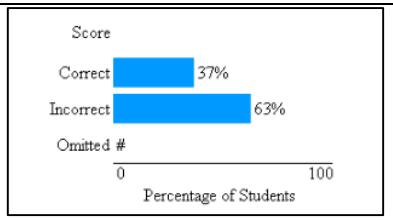
Understand similarity in terms of similarity transformations

- **G.SRT.2.** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

For more information on the MCCS- Grade Level Standards by Domain and Cluster, please visit: http://opi.mt.gov/Curriculum/montCAS/MCCS/index.php?gpm=1_4

Year	Grade	Block	#	Type	Difficulty	Content Area	% Correct	Item	Description	Iowa CCSS Code	Alaska CCSS Code
2003	8	6	19	MC	Hard	Number sense, properties, and operations	38.6	Item1	Use proportional reasoning to find the distance between two towns along a line	3.OA.2	7.RP.2
2003	8	7	6	SCR	Hard	Number sense, properties, and operations	24.52	Item2	Compare percent reduction (calculator available)	6.RP.3	7.RP.3.
2003	8	10	10	MC	Medium	Number sense, properties, and operations	59.77	Item3	Identify equivalent ratio	6.RP.3	7.RP.2
2005	8	3	17	MC	Hard	Number properties and operations	36.55	Item4	Solve a story problem involving percent increase	6.RP.3	7.RP.3.
2007	8	7	9	MC	Medium	Algebra	49.54	Item5	Use formula to solve a problem (calculator available)	6.EE.9	7.RP.1.
2007	8	9	7	MC	Easy	Number properties and operations	62.13	Item6	Convert raw points to a percentage (calculator available)	6.RP.3	7.RP.3.
2007	8	9	10	SCR	Medium	Algebra	56.13	Item7	Complete a table and write an algebraic expression (calculator available)	6.EE.9	7.RP.2
2007	8	11	19	MC	Hard	Number properties and operations	36.65	Item8	Determine distance given rate and time	6.EE.9	7.RP.1.
2009	8	10	5	MC	Easy	Number properties and operations	72.5	Item9	Determine a quantity based on given percent	6.RP.3	7.RP.3.
2011	8	8	6	SCR	Hard	Data analysis and probability	28.7	Item10	Express a numeric quantity as a percent (calculator available)	6.RP.3	7.RP.3.
2011	8	8	12	MC	Medium	Algebra	50.11	Item11	Evaluate equation for a given value in context (calculator available)	6.EE.1	7.RP.1.
2011	8	8	14	MC	Medium	Geometry	46.28	Item12	Compare similar parallelograms (calculator available)	G.SRT.2	7.RP.1.
2011	8	9	4	MC	Easy	Number properties and operations	68.27	Item13	Find wages earned (calculator available)	5.NBT.7	7.RP.2
#	#	#	#	#	#	#	#	Item14	SBAC Practice Item (8, 10, and 13)	#	7.RP.2.
#	#	#	#	#	#	#	#	Item15	SBAC Practice Item (3 and 16)	#	7.RP.3

NAEP Content Area: Algebra **Question:** Evaluate equation for a given value in context (calculator available). Gr.8. 2011. Item11 **Iowa CCSS classification**: 6.EE.1; **Alaska CCSS classification**: 7.RP.1.	**National Data:**	**MT Data:**
A reasonable prediction of the distance d in feet, that a car travels after the driver has applied the brakes can be found by using the formula $d=0.055r^2$, where r is the speed of the car in miles per hour. If Mario is driving at 60 miles per hour and applies the brakes, then according to the formula, how many feet will Mario's car travel before it stops? A. 330 B. 198 C. 10.89 D. 6.6 E. 3.3		**53% correct** Answer: B
NAEP Content Area: Algebra **Question:** Use formula to solve a problem (calculator available). Gr.8. 2007. Item5 **Iowa CCSS classification**: 6.EE.9; **Alaska CCSS classification**: 7.RP.1.	**National Data:**	**MT Data:**
The formula $d = 16\ t^2$ gives the distance d, in feet, that an object has fallen t seconds after it is dropped from a bridge. A rock was dropped from the bridge and its fall to the water took 4 seconds. According to the formula, what is the distance from the bridge to the water? A. 16 feet B. 64 feet C. 128 feet D. 256 feet E. 4,096 feet		**52% correct** Answer: D
NAEP Content Area: Geometry **Question:** Compare similar parallelograms (calculator available). Gr.8. 2011. Item12 **Iowa CCSS classification**: G.SRT.2; **Alaska CCSS classification**: 7.RP.1.	**National Data:**	**MT Data:**
 Parallelograms *ABCD* and *PQRS* above are similar. What is the length of side *QR*? A. 4.5 B. 9 C. 12 D. 15 E. 18		**48% correct** Answer: E
NAEP Content Area: Number properties and operations **Question:** Determine distance given rate and time. Gr.8. 2007. Item8 **Iowa CCSS classification**: 6.EE.9; **Alaska CCSS classification**: 7.RP.1.	**National Data:**	**MT Data:**

An airplane climbs at a rate of 66.8 feet per minute. It descends at twice the rate that it climbs. Assuming it descends at a constant rate, how many feet will the airplane descend in 30 minutes? A. 96.8 B. 133.6 C. 1,002 D. 2,004 E. 4,008		**46% correct** Answer: E

SBAC Practice Test Items, Item14

Alaska CCSS classification: 7.RP.2.

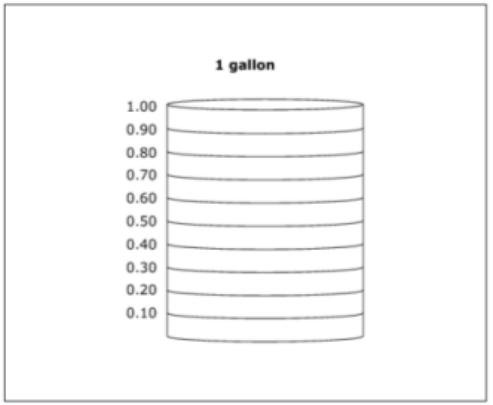
8

Tim makes 80 gallons of paint by mixing 48 gallons of green paint with 32 gallons of blue paint.

What part of every gallon is from green paint?

The model represents 1 gallon of mixed paint.

Select the bars to show how much of the gallon is from green paint.



10

Jerry needs 216 posts to build a fence. He has 88 posts and needs *p* more.

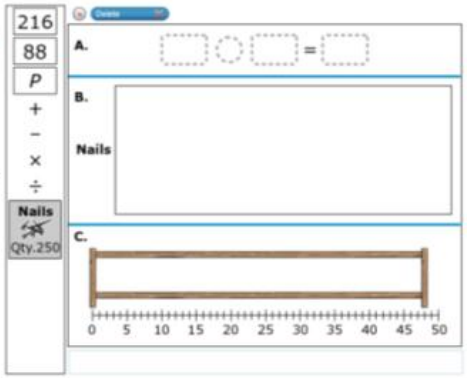
A. Drag numbers into the boxes and an operational symbol into the circle to create an equation to show how to solve for the number of posts Jerry needs.

Each post requires 8 nails for installation. There are 250 nails in a box.

B. Drag boxes of nails into the Answer Space to show how many boxes of nails Jerry needs.

Jerry will build another fence that is 48 feet long. The posts can be a minimum of 5 feet apart and a maximum of 9 feet apart. The posts should be equally spaced.

C. Click the number line to design a fence that uses the fewest posts possible.



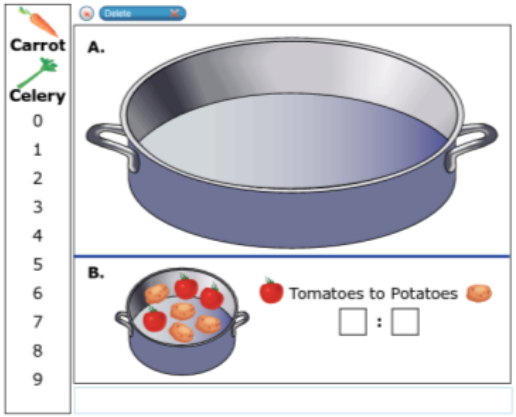
13

Scott's soup recipe for 4 servings has 3 carrots and 2 celery sticks.

A. Drag carrots and celery sticks into the pot to show how much Scott needs for 8 servings.

Paul's soup has tomatoes and potatoes.

B. Drag numbers into the boxes to show the ratio of tomatoes to potatoes.



Scoring Guide:

8. For this item, a full-credit response (1 point) includes:

- 0.6 gallon of green paint

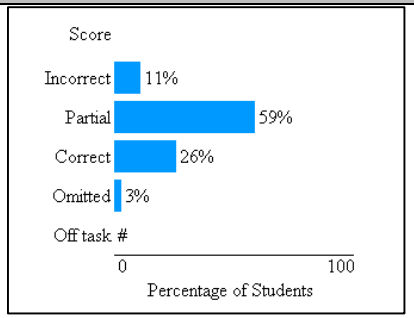
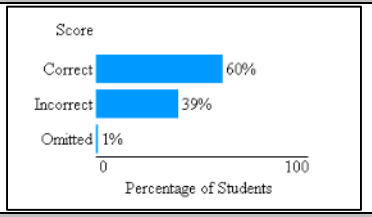
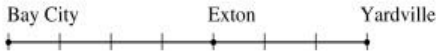
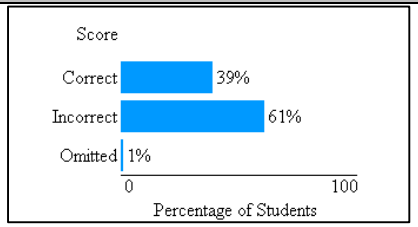
10. For this item, a full-credit response (3 points) includes:

- $p + 88 = 216$
  AND
- 7 boxes of nails
  AND
- 5 posts placed 8 feet apart

13. For this item, a full-credit response (2 points) includes:

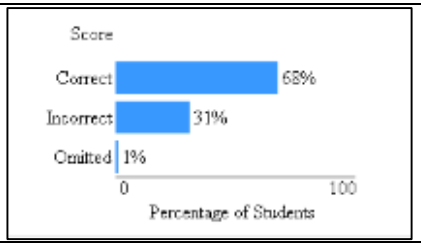
- 6 carrots and 4 celery sticks
  AND
- 3.4

NAEP Content Area: Algebra

Question: Complete a table and write an algebraic expression (calculator available). Gr.8. 2007. Item7 **Iowa CCSS classification**: 6.EE.9; **Alaska CCSS classification**: 7.RP.2	**Key/ Scoring Guide:**	**National Data:**	**MT Data:**
Sarah has a part-time job at Better Burgers restaurant and is paid $5.50 for each hour she works. She has made the chart below to reflect her earnings but needs your help to complete it. (a) Fill in the missing entries in the chart. Hours Worked / Money Earned (in dollars) 1 / $5.50 4 / / $38.50 $7\frac{3}{4}$ / $42.63 (b) If Sarah works *h* hours, then, in terms of *h,* how much will she earn?	**Sample Correct Responses:** Hours Worked / Money Earned (in dollars) 1 / $5.50 4 / $22.00 7 / $38.50 $7\frac{3}{4}$ / $42.63 5.5 x h = 5.5h		**28% correct** **60% partial**
NAEP Content Area: Number sense, properties, and operations **Question:** Identify equivalent ratio. Gr.8. 2003. Item3 **Iowa CCSS classification**: 6.RP.3; **Alaska CCSS classification**: 7.RP.2		**National Data:**	**MT Data:**
Which of the following ratios is equivalent to the ratio of 6 to 4? A. 12 to 18 B. 12 to 8 C. 8 to 6 D. 4 to 6 E. 2 to 3			**65% correct** Answer: B
NAEP Content Area: Number sense, properties, and operations **Question:** Use proportional reasoning to find the distance between two towns along a line. Gr.8. 2003. Item1 **Iowa CCSS classification**: 3.OA.2; **Alaska CCSS classification**: 7.RP.2		**National Data:**	**MT Data:**
 On the road shown above, the distance from Bay City to Exton is 60 miles. What is the distance from Bay City to Yardville? A. 45 miles B. 75 miles C. 90 miles D. 105 miles			**41% correct** Answer: D
NAEP Content Area: Number properties and operations **Question:** Find wages earned (calculator available). Gr.8. 2011. Item13 **Iowa CCSS classification**: 5.NBT.7; **Alaska CCSS classification**: 7.RP.2		**National Data:**	**MT Data:**

Last week Maureen earned $288.00 (before taxes) for working 40 hours. This week Maureen worked 29 hours at the same rate of pay. How much did Maureen earn (before taxes) this week?

A. $72.00
B. $72.50
C. $203.00
D. $208.80
E. $397.24



72% correct

Answer: D

SBAC Practice Test Items, Item15

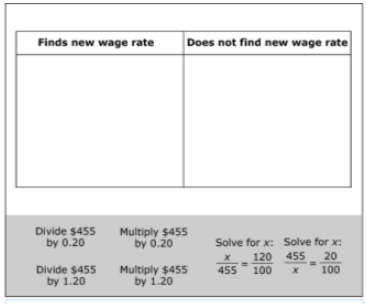
Alaska CCSS classification: 7.RP.2

3

George's weekly pay rate is $455 per week.
He receives a 20% raise.

How can George calculate his new weekly wage rate?

Drag each calculation to the category that correctly describes whether the calculation on its own can find George's new weekly pay rate.



For this item, a full-credit response (3 point) includes:

- "Multiply $455 by 1.20" and "Solve for x: $\frac{x}{455}=\frac{120}{100}$" in the "Finds new wage rate" column
  AND
- "Divide $455 by 0.20", "Divide $455 by 1.20", "Multiply $455 by 0.20", and "Solve for x: $\frac{455}{x}=\frac{20}{100}$" in the "Does not find new wage rate" column

For partial credit, the student

- correctly places 5 out of 6 responses (2 point)
  OR
- correctly places 4 out of 6 responses (1 point)

16

Mike does not want to spend more than $18.00 on a new collared shirt.

Select all of the following descriptions of prices for collared shirts that Mike would buy.

- ☐ 10% off $19.00
- ☐ 15% off $20.00
- ☐ 25% off $28.00
- ☐ $14.85, plus a $3.25 shipping fee
- ☐ $15.55, plus a $2.40 shipping fee
- ☐ $16.25, plus a $1.90 shipping fee

For this item, a full-credit response(1 point) includes:

- option A
  AND
- option B
  AND
- option E

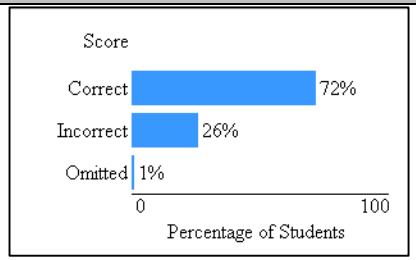
NAEP Content Area: Number properties and operations
Question: Determine a quantity based on given percent. Gr.8. 2009. Item9
Iowa CCSS classification: 6.RP.3; **Alaska CCSS classification**: 7.RP.3.

National Data:

MT Data:

The school carnival committee sold a total of 200 tickets for the grand prize drawing. Sue bought enough tickets so that she had a 20 percent chance of winning the grand prize. How many tickets did Sue buy?

A. 20
B. 40
C. 160
D. 400
E. 1,000



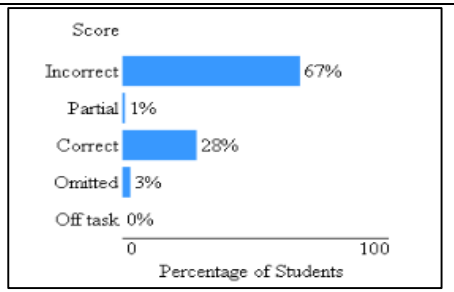
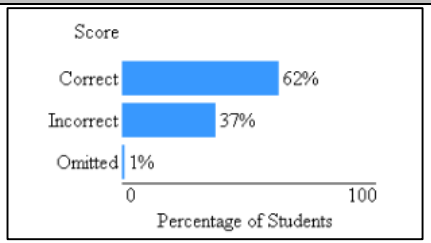
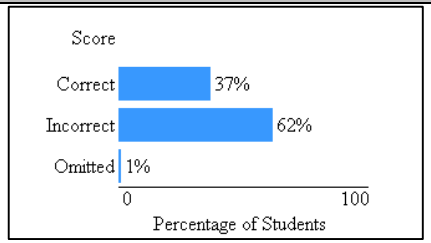
73% correct

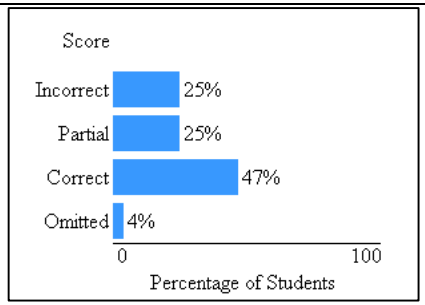
Answer: B

NAEP Content Area: Data analysis and probability
Question: Express a numeric quantity as a percent (calculator available). Gr.8. 2011. Item10
Iowa CCSS classification: 6.RP.3; **Alaska CCSS classification**: 7.RP.3.

National Data:

MT Data:

On average, thunder is heard in Tororo, Uganda, 251 days each year. What is the probability that thunder will be heard in Tororo on any day? (1 year = 365 days) Give your answer to the nearest percent. Answer: _____ %		**41% correct** **1% partial**
NAEP Content Area: Number properties and operations **Question:** Convert raw points to a percentage (calculator available). Gr.8. 2007. Item6 **Iowa CCSS classification**: 6.RP.3; **Alaska CCSS classification**: 7.RP.3.	**National Data:**	**MT Data:**
Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored? A. 0.91% B. 1.10% C. 52% D. 91% E. 95%		**71% correct** Answer: D
NAEP Content Area: Number properties and operations **Question:** Solve a story problem involving percent increase. Gr.8. 2005. Item4 **Iowa CCSS classification**: 6.RP.3; **Alaska CCSS classification**: 7.RP.3.	**National Data:**	**MT Data:**
There were 90 employees in a company last year. This year the number of employees increased by 10 percent. How many employees are in the company this year? A. 9 B. 81 C. 91 D. 99 E. 100		**44% correct** Answer: D
NAEP Content Area: Number sense, properties, and operations **Question:** Compare percent reduction (calculator available). Gr.8. 2003. Item2 **Iowa CCSS classification**: 6.RP.3; **Alaska CCSS classification**: 7.RP.3.	**Key/Scoring Guide:**	**National Data:**	**MT Data:**

One store, Price Pleasers, reduces the price each week of a $100 stereo by 10 percent of the original price. Another store, Bargains Plus, reduces the price each week of the same $100 stereo by 10 percent of the previous week's price. After 2 weeks, how will the prices at the two stores compare? Ⓐ The price will be cheaper at Price Pleasers. Ⓑ The price will be the same at both stores. Ⓒ The price will be cheaper at Bargains Plus. Explain your reasoning.	Correct response—cheaper at Price Pleasers with an explanation that compares price at each store after 2 weeks ($80 vs. $81) OR Cheaper at Price Pleasers with an explanation that generalizes as described in solution above NOTE: Score CORRECT if incorrect answer is B or C with a clear statement that Price Pleasers is cheaper and explanation is correct and complete. **Partial** Cheaper at Price Pleasers with anything less than a complete explanation OR Computes the correct amount for at least 2 weeks for either Price Pleasers or Bargain Plus, but conclusion is missing, incomplete, or incorrect (if the store is not identified the score is still a 2)		**48% correct** **23% partial**